

Sun Vector Calculation

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1 Context

This paper will explain the process used by STK to produce Sun Vector Positions within the software. This paper will first outline each step in the process and then will explain each step in detail. Additional references will be provided at the end

2 General Process Description

STK's Sun Vector calculation begins with determining the locations of all the objects involved (STK Object, its central body, and the Sun), relative to an inertial frame (ICRF) located at the solar system barycenter. General relativity effects are ignored, but special relativity (to first order in v/c) are considered. The locations of celestial bodies over time are taken from the high precision ephemeris generated by JPL. [Its N-body general relativity computation of ephemeris for the major celestial objects (Sun, planets, dwarf planets, and Earth's Moon) is updated every few years— the current version is DE-430.]

Other lower-fidelity algorithms exist that predict the locations of celestial bodies. David Vallado's book "Fundamentals of Astrodynamics and Applications" references a well known algorithm that is often used to calculate Sun Vectors. This algorithm is purported to have an accuracy of 0.01 from a ground location. Being lower fidelity, this algorithm does not yield the same results found in STK.

In order to produce sun vector calculations, STK uses a three step process to calculate highly accurate data.

1. JPL Ephemeris Data Implementation
2. Light Time Delay Calculation
3. Frame Transformation

2.1 JPL Ephemeris Data Implementation¹

The ephemerides for most central bodies are obtained from a Developmental Ephemeris (DE) file or a JPL SPICE file. Such files contain tables of position and velocity over a large span of time.

The Developmental Ephemeris is generated from a numerical integration of the solar system, using barycentric positions for the outer planets and including effects from general relativity. The ephemerides is made available to the public from a JPL website. AGI has collected the ephemerides for the period 1960-2050 and packaged it into a JPL DE file, a particular binary format that allows software (publicly available from JPL) to extract and interpolate the data. By default, version 430 (i.e., DE430) dated Feb 2008 will be used. It contains the then-best ephemerides of the major planets, the Moon, and Sun and updates the attitude of the Moon over earlier versions (DE-421, DE-405, DE-200).

JPL also makes available ephemerides in another format called SPICE. There are SPICE files that contain the same developmental ephemerides as normally shipped in a DE file. In addition, JPL makes available ephemerides for a wide range of celestial bodies in the solar system. These ephemerides are usually developed during JPL mission planning and operations. Time spans for different bodies can vary widely; moreover, there may be several versions of ephemerides for the same celestial body developed at different times (and possibly incorporating different sets of data). For example, the ephemerides for the Jovian system was updated and improved during the Galileo mission.

SPICE files are read and interpolated using SPICE software available from JPL. The SPICE toolkit natively reports the position of planetary centers, rather than the barycenters of planetary systems from a DE file. AGI has incorporated those parts of SPICE dealing with ephemeris interpolation into its software, and collected the best ephemerides for many of the larger bodies of the solar system into a few SPICE files. Note that JPL does not publish a solar system SPICE file of its own.

Currently, all central bodies shipped with AGI software could be configured to use the ephemerides from SPICE files. However, the older DE file format provides better time precision when requesting interpolated ephemeris. Thus, the Earth, Moon, and Sun are configured to use the older DE file format as their ephemeris source; all other bodies (except for Ceres) use ephemerides contained in SPICE files. Ceres uses a simple analytic formula (essentially a two-body formula where the elements are modeled as drifting linearly in time).

All central body ephemerides span the period 1990 – 2048 (at least), except for the Jovian system which spans 2000-2048.

2.2 Light Time Calculations

From the Ephemeris, THE Sun-Earth Vector is calculated by taking into account light time delay and aberration. Simply put, Light Time Delay assumes the

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speed of light to be a finite value. As such, the time it takes for light to travel from point A to point B needs be considered.

2.2.1 Transmission from Point A to Point B

Let \mathbf{R}_A locate object A in inertial frame F and let the relative position vector \mathbf{r} be defined by:

$$\mathbf{r}(t) = \mathbf{R}_B(t + \Delta t) - \mathbf{R}_A(t) \quad (1)$$

Where t is the time of transmission from A and $t + \Delta t$ is the time of reception at B. The light time delay is Δt which will depend on t as well. Let $r = |\mathbf{r}|$ be the range between the objects. Then $\Delta t = \frac{r}{c}$. One usually knows the locations for the objects A and B and computes the light time delay at time t through iteration. First, a value of Δt is guessed (often taken to be 0.0 or the last value computed at a previous time) and $\mathbf{r}(t)$ is computed. A new value for Δt is found from r/c and the procedure repeats. The iteration stops whenever the improvement in the estimate to Δt is less than the light time delay convergence tolerance. Typically, few iterations are required as the procedure converges very rapidly.

Inertial Frame choice is important for calculations. For more information on inertial frame choice, visit [Light Time Delay and Apparent Position, pg. 5](#).

2.3 Aberration

Aberration is the change in the perceived direction of motion caused by the observer's own motion. There are three kinds of aberration accounted for within STK's calculations. They are as follows:

1. Stellar
2. Planetary
3. Annual

2.3.1 Stellar Aberration

Aberration caused by an observer's motion in the solar system as the observer receives the light can be computed, and is referred to as stellar aberration (transmission time at the star is too uncertain to allow for light time delay to be considered). Let the direction to a star from an observer (accounting for proper motion of the star and parallax) be $\hat{\mathbf{e}}_r$. Then the apparent direction of the star, accounting for stellar aberration, is²:

$$\hat{\mathbf{p}} = \frac{\hat{\mathbf{e}}_r + \boldsymbol{\beta}}{\|\hat{\mathbf{e}}_r + \boldsymbol{\beta}\|} \quad (2)$$

²This formula models the observer receiving a signal, not transmitting a signal

where:

$$\beta = \frac{\mathbf{v}}{c} \quad (3)$$

and \mathbf{v} is the velocity of the observer with respect to the solar system barycenter frame. This formula is the Galilean formula. To find the Special Relativity equation, which is a use of the Lorentz transformation for velocities, please visit [Light Time Delay and Apparent Position, pg. 7](#) and navigate to the citations given.

2.3.2 Planetary Aberration

Planetary aberration refers to the combination of light time delay and stellar aberration and, to the order β , can be computed using simplified Galilean formulas. The proper apparent direction is computed to be:

$$\mathbf{r}_p = \mathbf{r} - \sigma \Delta t \mathbf{v} = c \Delta t (\hat{\mathbf{e}}_{\mathbf{r}} - \sigma \beta) = \mathbf{R}_B(t + \sigma \Delta t) - \mathbf{R}_A(t) - \sigma \Delta t \frac{d\mathbf{R}_A}{dt} \quad (4)$$

where $\sigma = 1$ when modeling a signal transmitted from A and $\sigma = -1$ when modeling a signal received at A. \mathbf{r} is the apparent relative position of B with respect to A, so that the light path range r is $c \Delta t$ and $\beta = \frac{v}{c}$. The vector \mathbf{r}_p is the proper apparent relative position of B with respect to A, where the term ‘proper’ indicates that this quantity is computed as perceived by A (really, by an observer at A moving in a co-moving inertial frame).

Eventually, this equation can be brought to the following form³

$$\hat{\mathbf{p}} = \frac{\hat{\mathbf{e}}_{\mathbf{r}} - \sigma \beta}{\|\hat{\mathbf{e}}_{\mathbf{r}} - \sigma \beta\|} \quad (5)$$

2.3.3 Annual Aberration

Total Aberration identifies the contribution of the observer’s central velocity in the solar system:

$$\mathbf{v} = \mathbf{v}_{cb} + \mathbf{v}_{A/cb} \quad (6)$$

where \mathbf{v}_{cb} is the velocity of the central body with respect to the solar system barycenter and $\mathbf{v}_{A/cb}$ is the velocity of the observer A with respect to the central body. When only \mathbf{v}_{cb} is used, only the annual aberration effects have been considered.

³For a walk through of the steps taken to reach this conclusion visit [Light Time Delay and Apparent Position, pg. 9](#)

2.4 Frame Transformation

Special relativity effects must be computed in an appropriate inertial frame, where light is modeled to move along straight lines. The ICRF frame, with origin at the solar system barycenter, is used as the light propagation frame. Once computed in this frame, the Sun-Earth vector can be transformed into any other frame. In STK, the transformation is done without using Special Relativity (i.e., Galilean transformations are employed).

For more information on Central Body Reference Frames visit [Technical Overview of Central Body Reference Frames](#)